# The h-index of a conglomerate

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#### Abstract

The h-index of a conglomerate is defined. It is further shown how Prathap's h-indices can be presented in a conglomerate framework. An example illustrates the use of the conglomerate framework.

Keywords: *h*-index; conglomerates; Prathap's institutional h-indices

# The conglomerate framework (Rousseau, 2005)

If a scientist's publications are ranked in decreasing order of number of citations, then his/her lifetime achievement *h*-index is the highest rank such that the first *h* publications each received *h* or more citations. It is well-known that the idea of an *h*-index can be applied to many source-item relations (Egghe & Rousseau, 2006). Here we present a definition in the general framework of a conglomerate.

A conglomerate, introduced in (Rousseau, 2005), is a framework for informetric (and other) research. Figure 1 illustrates the basic elements of a conglomerate. It consists of two collections and two mappings. The first collection is a finite set, denoted as S, and called the source collection. Its elements are called sources. The second collection, denoted as P, is called the pool. It is not necessarily finite, but in practical applications it will always be finite. Further a mapping f is given from S to  $2^P$ , the set of all subsets of P. For each  $S \subseteq S$ , f(S) is a subset of P, called the item-set of S. The union of all P in P belonging to at least one item-set is called the item collection, denoted as  $I \subseteq P$ . The map f itself is called the source-item map.

Each set f(s) is mapped to a number, called the magnitude of this set. This mapping is denoted as m and maps  $f(s) \in 2^P$  to  $m(f(s)) \in \mathbb{R}^+$  (referred to as the m-value of

source s). The mapping itself is called the magnitude function. In simple cases m is the counting measure which maps f(s) to the number of elements in f(s). The conglomerate is a quadruple C = (S, P, f, m).

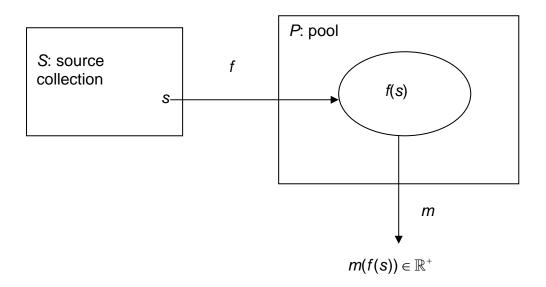


Fig. 1 Schematic representation of a conglomerate taken from (Rousseau, 2005)

These steps lead to a first important element in informetric research, namely the ratio of the sum of all magnitudes of item-sets, and the number of elements in the source collection. In a general setting this ratio is referred to as the conglomerate ratio:

Conglomerate ratio = 
$$\frac{\sum_{s \in S} m(f(s))}{\#(source\ collection)}$$

In concrete cases this conglomerate ratio is e.g. a journal's impact factor. Finally, the source-item relation of a conglomerate leads to three lists. The first one just consists of all sources and the magnitude of their corresponding item sets. The second list is the same as the first one, but sources are ranked in decreasing order of the magnitudes of their corresponding item-sets. We will refer to this list as a Zipf list and the rank of a source in this list is called its Zipf rank. The first list can also, if desired, be rewritten in size-frequency form, leading to a third list associated with the source-item relation of a conglomerate. We may refer to such a list as a Lotka list.

## The h-index of a conglomerate

Based on Hirsch's original idea (Hirsch, 2005) we now define the *h*-index of a conglomerate.

## Definition

The h-index of conglomerate C is defined as the highest rank such that the magnitude corresponding to Zipf rank h is at least equal to h. The set of these first h sources is called the h-core. As usual provisions must be made in case of ties at rank h.

We note that not each conglomerate leads to a meaningful *h*-index, or meaningful conglomerate ratio for that matter.

The standard lifetime *h*-index is the *h*-index without any kind of correction for self-citations or co-authorships. Here, the Web of Science (WoS) is used as an example but the procedure is completely equivalent for Scopus or any other citation database. Scientist A's standard lifetime *h*-index, as determined from the WoS, is obtained by taking *S* equal to all articles (co-)authored by A, and collection *P* as all articles included in the WoS. The function *f* maps each article to the set of articles citing this article; *m* is just the counting function, stating how many citations each article has received. Finally, the lifetime *h*-index of scientist A is derived from the corresponding Zipf list. As described here the h-index includes citations to articles outside the group of WoS (or Scopus) journals. By restricting the set S to all articles published in journals covered by the WoS (or Scopus) one obtains the h-index which is usually mentioned. The conglomerate ratio is here equal to the average number of citations received in the WoS (or Scopus) by scientist A.

Most common indicators, including the h-index and the impact factor, can be determined without considering all elements in the pool (the set P), which would be very impractical for large databases. Generally, one only uses the item collection.

Once a Zipf list is obtained it is not difficult to define a g-index (Egghe, 2006) or an R-index (Jin et al., 2007) for a general conglomerate.

## Prathap's $h_1$ - and $h_2$ -indices presented in a conglomerate framework

In a brief letter published in *Current Science*, Prathap (2006) proposed using two different types of h-indices for institutional evaluations: a level one h-index ( $h_1$ ) and a level two h-index ( $h_2$ ), where level one h-index is equal to  $h_1$  if the institution (this is the collection of all its researchers) has published  $h_1$  papers, each of which has at least  $h_1$  citations; and its level two h-index is  $h_2$  if the institution has  $h_2$  researchers, each having an individual h-index which is at least equal to  $h_2$ . If an institute has just one or two high level scientists then the institute's  $h_1$  value will be high but its  $h_2$  will be very low. Another institute that has many scientists of high quality may have approximately the same  $h_1$  but a much higher  $h_2$ . In this way the combination of  $h_1$  and  $h_2$  yields useful information about the institute's research structure.

How can  $h_1$  and  $h_2$  be presented in a conglomerate framework? This is not difficult for  $h_1$ . As source collection we take the list (ranking plays no role here, so this may be an alphabetical or chronological list) of all articles on which at least one member of the institute has contributed during a given period. This list is denoted as IL (institutional list). As pool P we take any appropriate (local, regional or international) citation database D. The map  $f_1$  maps each article a in IL, to the set of articles in D citing this

article. As usual  $m_1$  is the counting measure, so  $m_1(f_1(a))$  is the number of citations received by article a in D during a given citation period. We determine the institutional h-index  $h_1$  from the corresponding Zipf list. The conglomerate ratio for this conglomerate is the average number of citations (per article) received by articles written by scientists of this institute.

Describing  $h_2$  in the conglomerate framework is somewhat more complicated. Now we use as source collection the set of all the institute's scientists, denoted as SC. Each scientist (s) is mapped to a set of pairs. The first element of such a pair is an article  $a_{(s)}$  written by scientist s, hence an article belonging to IL. The second element of this pair is the set of all articles in D citing article  $a_{(s)}$ , during the period under study. Hence an image f(s) looks like:

$$\left\{\left(a_{1(s)},\left\{c_{1}^{(1)},c_{2}^{(1)},...,c_{j}^{(1)}\right\}\right),\left(a_{2(s)},\left\{c_{1}^{(2)},c_{2}^{(2)},...,c_{k}^{(2)}\right\}\right),\left(a_{3(s)},\left\{c_{1}^{(3)},...,c_{n}^{(3)}\right\}\right),...\right\},\right.$$

where  $a_{1(s)}, a_{2(s)}, \ldots$  are articles (co)authored by scientist s, and the  $c_j^{(y)}, j=1,2,\ldots$  denote the different citing articles of the y<sup>-th</sup> article written by scientist s. Each a belongs to IL, while each set of citing articles belongs to  $2^D$ . The pool corresponding to this situation is IL ×  $2^D$ . Hence the source-item map  $f_2$  maps SC to  $2^{IL \times 2^D}$  (the set of subsets of IL ×  $2^D$ ). The magnitude function of this conglomerate maps, for each scientist s, the associated set of pairs to this scientist's h-index, denoted as h(s). Observe that each image, f(s), contains exactly the information needed to determine a scientist's h-index. Now the Zipf list associated to this conglomerate is the ranked list of all the h-indices of the institute's scientists. It naturally leads to  $h_2$ . The conglomerate ratio of the second conglomerate is the average h-index of all scientists belonging to this institute. In order to clarify this construction we have added a simple fictitious example in the appendix.

This section shows how Prathap's h-indices can be described in a conglomerate framework. As the construction of  $h_2$  is a special case of the construction of successive h-indices (Schubert, 2007) this construction also shows how successive h-indices can be described in a conglomerate framework.

#### Conclusion

It is shown how the conglomerate framework is the natural environment for the study of h-indices. This definition leads to a huge generalization of the original concept. It is shown how Prathap's institutional h-indices can be presented in this framework. Finally, it is mentioned that once the Zipf list of a conglomerate is drawn, other h-type indices such as Egghe's g and the R-index can also be generalized to the conglomerate framework

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# **Appendix**

A fictitious example: The SMALL-IS-BEAUTIFUL Institute

The SMALL-IS-BEAUTIFUL Institute is a research institute with only four scientists. Their publication-citation record over the investigated period is given in Table 1.

Table 1

SC	IL	Citing articles
SC1	ART <sub>SC1</sub> 1	Citingarticle1
		Citingarticle2
		Citingarticle3
	ART <sub>SC1</sub> 2	Citingarticle1
		Citingarticle3
	ART <sub>SC1</sub> 3	
SC2	ART <sub>SC2</sub> 1	Citingarticle4
SC3	ART <sub>SC3</sub> 1	Citingarticle5
	ART <sub>SC3</sub> 2	Citingarticle6
		Citingarticle7
		Citingarticle8
		Citingarticle9
		Citingarticle10
SC4	ART <sub>SC4</sub> 1	
	ART <sub>SC4</sub> 2	Citingarticle6
	ART <sub>SC4</sub> 3	Citingarticle6
		Citingarticle7
		Citingarticle8
	ART <sub>SC4</sub> 4	Citingarticle6
		Citingarticle7
		Citingarticle8
		Citingarticle11

The second column contains IL and we assume that a certain P has been chosen from which the citing articles have been retrieved.

The first source-item map  $f_1: IL \rightarrow 2^P$  is defined as follows:

```
Then m_1(f_1(ART_{SC1}1)) = 3
         m_1(f_1(ART_{SC1}2)) = 2
         m_1(f_1(ART_{SC1}3)) = 0
         m_1(f_1(ART_{SC2}1)) = 1
         m_1(f_1(ART_{SC3}1)) = 1
         m_1(f_1(ART_{SC3}2)) = 5
         m_1(f_1(ART_{SC4}1)) = 0
         m_1(f_1(ART_{SC4}2)) = 1
         m_1(f_1(\mathsf{ART}_{\mathsf{SC4}}3)) = 3
         m_1(f_1(ART_{SC4}4)) = 4
This leads to the Zipf list
         ART<sub>SC3</sub>2
                           5
         ART<sub>SC4</sub>4
                           4
         ART<sub>SC1</sub>1
                           3
         ART<sub>SC4</sub>3
                           3
                           2
         ART<sub>SC1</sub>2
         ART<sub>SC2</sub>1
                           1
         ART<sub>SC3</sub>1
                           1
         ART<sub>SC4</sub>2
                           1
         ART<sub>SC1</sub>3
                           0
         ART<sub>SC4</sub>1
                           0
and hence h_1 = 3.
The second source-item map f_2 : SC \rightarrow 2^{lL \times 2^{D}} is defined as follows:
SC1 \rightarrow \{ (ART_{SC1}1, \{ Citingarticle1, Citingarticle2, Citingarticle3 \} ),
              (ART<sub>SC1</sub>2, { Citingarticle1, Citingarticle3 }),
              (ART_{SC1}3, \emptyset) }
SC2 \rightarrow \{ (ART_{SC2}1, \{ Citingarticle4 \}) \}
SC3 \rightarrow \{ (ART_{SC3}1, \{ Citingarticle5 \}), \}
              (ART<sub>SC3</sub>2, { Citingarticle6, Citingarticle7, Citingarticle8,
                                Citingarticle9, Citingarticle10 }) }
SC4 \rightarrow \{ (ART_{SC4}1, \emptyset), 
              (ART<sub>SC4</sub>2, { Citingarticle6 }),
              (ART<sub>SC4</sub>3, { Citingarticle6, Citingarticle7, Citingarticle8 }),
              (ART<sub>SC4</sub>4, { Citingarticle6, Citingarticle7,
                                Citingarticle8, Citingarticle11 }) }
Then m_2(f_2(SC1)) = 2 = h(SC1)
         m_2(f_2(SC2)) = 1 = h(SC2)
         m_2(f_2(SC3)) = 1 = h(SC3)
         m_2(f_2(SC4)) = 2 = h(SC4)
```

This leads to the Zipf list

SC1	2
SC4	2
SC2	1
SC3	1

and hence  $h_2 = 2$ .