Optimization of Tsukamoto Fuzzy Inference System using Fuzzy Grid Partition

Hartono

University of Sumatera Utara
Medan, 20214, Indonesia

Abstract - This research paper introduces a novel statement for optimizing Tsukamoto fuzzy inference system. Suppose we are given some mathematical programming problem that will solve using Tsukamoto Fuzzy Inference System. We will using the linkable label of the variable that construct from the Fuzzy Grid Partition in correcting the limitation of Tsukamoto Fuzzy Inference System. Our research will show the crisp value from the Tsukamoto Fuzzy Inference System and the crisp value from the process of optimization using Fuzzy Grid Partition. Our research will find a fair optimal solution to the original fuzzy problem.

Keywords – *Tsukamoto Fuzzy Inference System, Fuzzy Grid Partition, Linkable Label, Optimization.*

1. Introduction

Fuzzy logic is determined as a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic [1]. Fuzzy systems are a part of soft computing that works on the discipline of vagueness and gives results in an interpretable manner. Fuzzy system makes use of fuzzy set theory, fuzzy reasoning and inference mechanism so that such systems can be employed in various applications. In classical set theory an object can either a member of a given set or not while fuzzy set theory allows an object to belong to a set with a certain degree. Fuzzy system models fuzzy boundaries of linguistic terms by introducing gradual membership [2].

Tsukamoto fuzzy inference system are solving the problem in If-Then Rules Form. In Tsukamoto Method, each consequence of If-Then Rules must be represented by a fuzzy set with monotonous membership function. Consequently, the interference outputs of each rule are crisply presented in line with α-predicate [3]. In our earlier works we find some limitation of Tsukamoto Fuzzy Inference System in producing the Crisp Value. The limitation comes from the rules that we use in the Tsukamoto Method.

Fuzzy grid partition can determine the number of fuzzy rules comprising the underlying model as well. In Fuzzy Grid Partition, we can make some linkable label of variable for producing a linkable rule. There are three types of fuzzy partitions: 1) the grid partition, 2) the tree partition, and 3) the scatter partition [4].

The Grid Partition is the most commonly used fuzzy partitioning methods in practice (particularly in control applications). Wang and Mendel has used this type of fuzzy partition in their procedure for fuzzy rule extraction from numerical data [5].

In this research we will use the Fuzzy Grid Partition in producing linkable rules from the linkable label of the variable for optimizing the Tsukamoto Fuzzy Inference System .formulas in italic type, with subscripts and superscripts in a slightly smaller font size. This is acceptable.

2. Literature Review

2.1 Working of Fuzzy Logic

Fuzzy logic consists of three main structure: fuzzification, inference engine, and defuzzification [6]. The fuzzy logic structure can be seen in Figure 1.

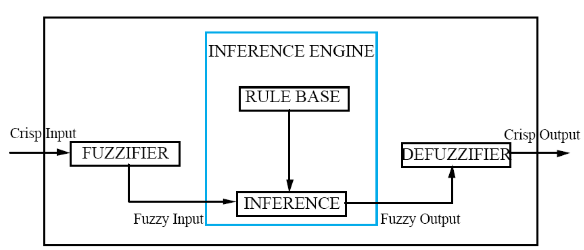

Fig. 1 Fuzzy Logic Process [6]

1) Crisp Input

The crisp input is always a numerical value limited to the universe of discourse [7].

2) Rule Base

Fuzzy reasoning includes two distinct parts: evaluating the rule antecedent (the IF part of the rule) and implication or applying the result to the consequent(the THEN part of the rule).

In classical rule-based systems, if the rule antecedent is true, then the consequent is also true. In fuzzy systems, where the antecedent is a fuzzy statement, all rules fire to some extent, or in other words they fire partially. If the antecedent is true to some degree of membership, then the consequent is also true to that same degree [7].

3) Inference

Fuzzy inference can be defined as a process of mapping from a given input to an output, using the theory of fuzzy sets [7].

2.2 Tsukamoto Fuzzy Inference System

The use of fuzzy sets provides a basis for a systematic way for the manipulation of vague and imprecise concepts. In particular, we can employ fuzzy sets to represent linguistic variables. A linguistic variable can be regarded either as a variable whose value is a fuzzy number or as a variable whose values are defined in linguistic terms [8].

Consider the following fuzzy inference system:

R_1: If X_1 is A_{11} and ... And X_n is A_{1n} then z is C_1
...
R_m: If X_1 is A_{m1} and ... And X_n is A_{mn} then z is C_m
Input: x_1 is y_1 and ... and x_n is y_n

Output:
The procedure for obtaining the crisp output from the crisp input vector $y = \{y_1, ..., y_n)$ and fuzzy rule-base $R = \{R_1, ..., R_m)$ consists of the following three steps [9].

1. We find the firing level of the ith rule as

$$\alpha_i = T\,(A_{i1}\,(Y_1), ..., A_{in}\,(y_n)),\ i = , ..., m \quad (1)$$

Where T usually is the minimum or the product t-norm

2. We determine the crisp output of the ith rule, denoted by z_i, from the equation $\alpha_i = C_i(Z_i)$ that is

$$z_i = C^{-1}{}_1\,(\alpha_i),\ i = 1,..., m \quad (2)$$

3. The overall system output is defined as the weighted average of the individual outputs, where associated weights are the firing levels. That is,

$$Z_0 = \frac{\alpha_1 Z_1 + ... + \alpha_m Z_m}{\alpha_1 + ... + \alpha_m} \quad (3)$$

The illustration of Tsukamoto's Inference Mechanism can be seen in Figure 2.

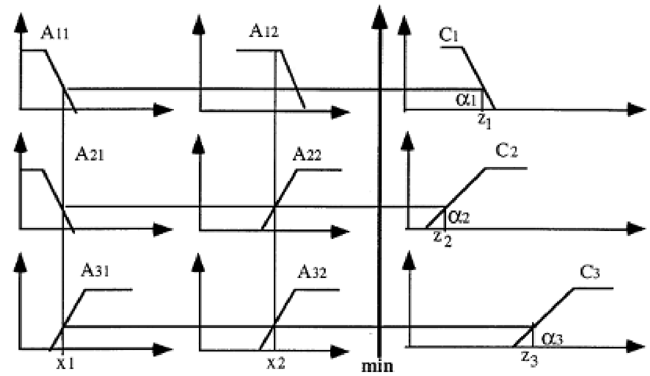

Fig. 2 Illustration of Tsukamoto's Inference System [8]

2.3 Fuzzy Grid Partition

Fuzzy grid partition can determine the number of fuzzy rules comprising the underlying model as well. In Fuzzy Grid Partition, we can make some linkable label of variable for producing a linkable rule.

There are three types of fuzzy partitions:
1) the grid partition,
2) the tree partition, and
3) the scatter partition [4].

The various method of fuzzy partition can be seen in Figure 3.

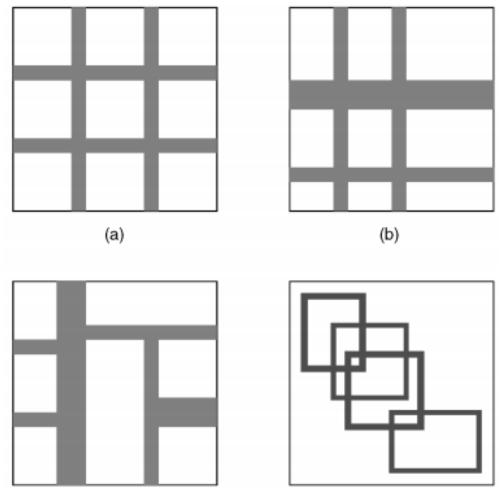

Fig. 3 Various methods for partitioning the input space: (a) gridpartition (static); (b) grid partition (adaptive); (c) tree partition; (d) scatter partition [4].

3. Research Design

3.1 Concepts of Linked Label

Assumes that we have 6 (six) rule like following rules [10]:

Rule-1: If X1 is Small and X2 is large the C3
Rule-2: If X1 is Small and X2 is Small then C1
Rule-3: If X1 is large and X2 is Medium then C4
Rule-4: If X1 is large and X2 is large then C2
Rule-5: If X1 is medium and X2 is small then C1
Rule-6: If X1 is medium and X2 is medium then C2

The Partition Form of that rules can be seen in Figure 4.

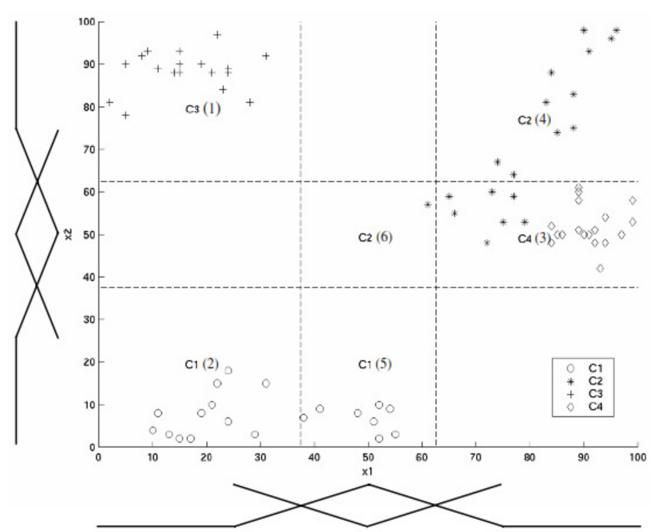

Fig. 4 Partition Form of The Rules [10]

Form Fig. 4 we can take some Preliminary Definitions [10].

Definition 1
Two linguistic labels of an input variable k, $l_{k,a}$ and $l_{k,b}$ with $a < b$, are said to be ***linkable*** when, if there are two rules that differ only in the label of the input k and being this label comprised between the beginning of the support of $l_{k,a}$ and the end of $l_{k,b}$ their consequents are the same. Mathematically, Iff:

$$\nexists\; r_p, r_q \in R \mid \text{if } r_p \equiv (l_{1,s_1}, \dots, l_{k,i}, \dots, l_{N,s_N}) \rightarrow c_p$$
$$\text{and } r_q \equiv (l_{1,s_1}, \dots, l_{k,j}, \dots, l_{N,s_N}) \rightarrow c_q$$
(4)

It happens that $c_p \neq c_q$ and $a \leq i \leq j \leq b$

Definition 2
Two rules r_p and r_q :
$$r_p \equiv (l_{1,p_1}, \dots, l_{k,p_k}, \dots, l_{N,p_N}) \rightarrow c_p$$
$$r_q \equiv (l_{1,q_1}, \dots, l_{k,q_k}, \dots, l_{N,q_N}) \rightarrow c_q$$
(5)
Are said to be ***redundant*** iff:
$c_p = c_q$ i and ($l_{k,pi}$ and $l_{k,qk}$) are linkable or equal k

Definition 3
Two linkable labels $l_{k,a}$ and $l_{k,b}$ are said to be ***linked*** or to ***constitute a link*** when they are linked definitively after eliminating a redundant rule.

3.2 Limitation of Tsukamoto Fuzzy Inference System

Suppose that we have the following problem that will be solved by the Tsukamoto Fuzzy Inference System.

1. Company A want to produce ABC Food. Data from the last month are as followings,

   - The biggest demands are 6500 packages/day, and the smallest demands are 2300 packages/day.
   - The biggest stock in the warehouse are 590 packages/day and the smallest stock in the warehouse are 120 packages/day

2. With the limitation, the company only can produce 7150 packages/day, and for the efficency the smallest produce are 2000 packages/day

   The process of production are using that 4 (four) rules like this:
   Rule-1: If demands is decrease and The Stock are many then production is decrease

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Rule-2: If demands is decrease and the stock is slightly then production is decrease
Rule-3: If demands is increase and the stock are many then production is increase
Rule-4: If demand is increase and the stock is slightly then production is increase

The question is how many product must be produced if there are demands 3900 packages and the stock in the warehouse is 310 packages?

We can use the membership function that can be seen in Figure 5.

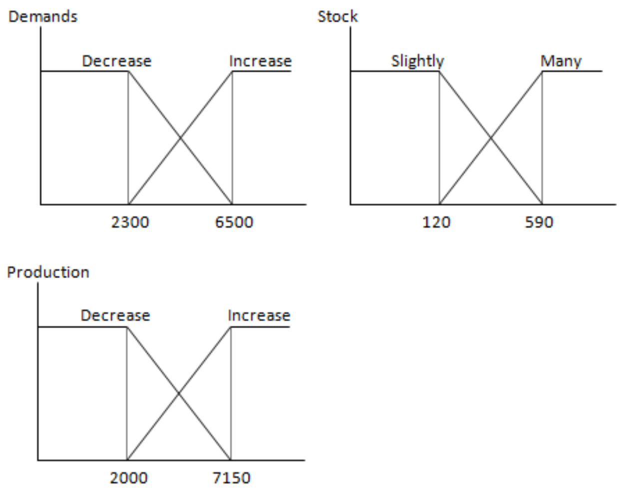

Fig. 5 Membership Functions of Demands, Stock, and Production

With the Tsukamoto Fuzzy Inference System we can calculate the number of product that must be producedd like this.

µDemandDecrease = (6500-3900) / 4200 = 0.62
µDemandIncrese = (3900-2300) / 4200 = 0.38
µStockSlightly = (590-310) / 470 = 0.6
µStockMany = (310-210) / 470 = 0.21

Rule-1:
α-1 = µDemandDecrease ∩ µStockMany
= 0.21
Rule-2:
α-2 = µDemandDecrease ∩ µStockSlightly
= 0.6
Rule-3:
α-3 = µDemandIncrease ∩ µStockMany
= 0.21
Rule-4:
α-4 = µDemandIncrease ∩ µStockSlightly
= 0.38

Rule-1: Consequent = Production is decrease
(7150-Z1) / 5150 = 0.21
7150-Z1 = 1081.5
Z1 = 6068.5

Rule-2: Consequent = Production is decrease
(7150-Z2) / 5150 = 0.6
7150-Z2 = 3090
Z2 = 4060

Rule-3: Production is increase
(Z3 – 2000) / 5150 = 0.21
Z3 – 2000 = 1081.5
Z3 = 3081.5

Rule-4: Production is increase
(Z4 – 2000) / 5150 = 0.38
Z4 – 2000 = 1957
Z4 = 3957

Z = ((0.21 x 6068.5) + (0.6 x 4060) + (0.21 x 3081.5) + (0.38 x 3957)) / (0.21 + 0.6 + 0.21 + 0.38)
= (1274.4 + 2436 + 647.1 + 1503.66) / 1.4
= 5861.6 / 1.4 = 4186.54

Let we see, from the problem, we have produce: 4186.54 products, we have demands = 3900, and stock = 310. So, the number of stock after the production is 4186.54 – 3900 + 310 = 596.54

In the condition we can see that maximum stock in the warehouse that be allowed are 590. This is a limitation of the Tsukamoto Fuzzy Inference System.

3.3 Optimization of Tsukamoto Fuzzy Inference System

We recommend using Fuzzy Grid for the correction of Tsukamoto Fuzzy Inference System that can be seen in Figure 6.

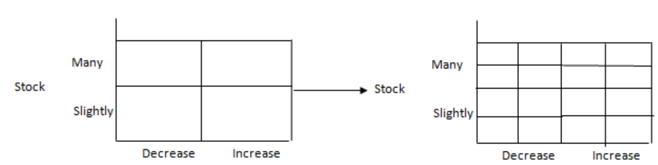

Fig. 6 Fuzzy Grid Partition of The Rule

According to Equation 4, in **Definition 2,** we can make a linkable label from Fuzzy Grid Partition.
Demand Decrease become Very Low, Decrease
Demand Increase become Increase, Very High

The membership function Demand as the result of Fuzzy Grid Partition can be seen in Figure 7.

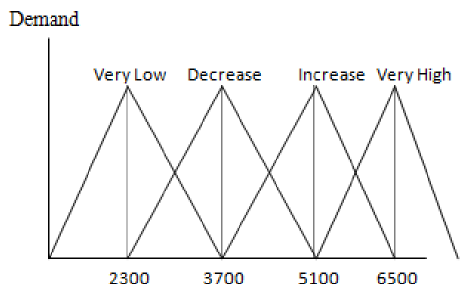

Fig. 7 Membership Function Demand as the Result of Fuzzy Grid Partition

Stock Slightly, the result of Fuzzy Grid Partition become Very Low, Slightly
Stock Many, the result of Fuzzy Grid Partition become Many, Very High

The membership function Stock as the result of Fuzzy Grid Partition can be seen in Figure 8.

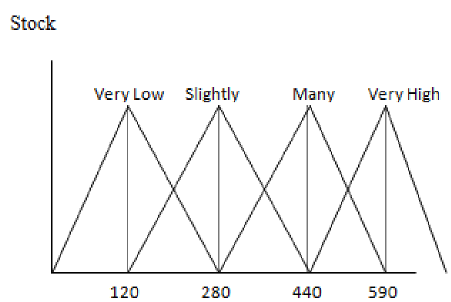

Fig. 8 Membership Function Stock as the Result of Fuzzy Grid Partition

Production decrease, the result of Fuzzy Grid Partition become Very Low, Decrease
Production Increase, the result of Fuzzy Grid Partition become Increase, Very High

The membership function Production as the result of Fuzzy Grid Partition can be seen in Figure 9.

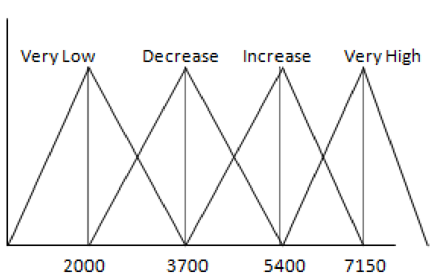

Fig. 9 Membership Function Production as the Result of Fuzzy Grid Partition

The result from this process, we can make a calculation like this.
µDemandDecrease = (5100-3900) / 1400 = 0.85
µDemandIncrease = (3900-3700) / 1400 = 0.14
µStockSlightly = (440-310) / 160 = 0.8125
µStockMany = (310-280) / 160 = 0.1875

Rule 1:
α-1 = µDemandDecrease ∩ µStockMany
= 0.1875
Rule 2:
α-2 = µDemandDecrease ∩ µStockSlightly
= 0.8125
Rule 3:
α-3 = µDemandIncrease ∩ µStockMany
= 0.14

Rule 4:
α-4 = µDemandIncrease ∩ µStockSlightly
= 0.14

(3700 – Z1)/1700 = 0.1875
Z1 = 3381.25

(5400 – Z2) / 1700 = 0.8125
Z2 = 4018.75

(5400 – Z3) / 1700 = 0.14
Z3 = 5162

Z4 = Z3 = 5162

Z = ((3381.25 x 0.1875) + (4018.75 x 0.8125) + (0.14 x 5162) + (0.14 x 5162)) / (0.1875 + 0.8125 + 0.14 + 0.14)
Z = (633.98 + 3265.23 + 722.68 + 722.68) / 1.28
Z = 5344.57 / 1.28
Z = 4175.44

Let we see, from the problem, we have produce: 4175.44 products, we have demands = 3900, and stock = 310. So, the number of stock after the production is 4175.44 – 3900 + 310 = 585.44

In the condition we can see that maximum stock in the warehouse that be allowed are 590. The limitation of Tsukamoto Fuzzy Inference System has been corrected.

4. Discussion

We have made a testing about the usage of Fuzzy Grid Partition in correcting the limitation of Tsukamoto Fuzzy System. Our simple mathematical problem, has shown us about the limitation of Tsukamoto Fuzzy Inference System. Using linkable label, we can make the linkable label in the membership function of every label. We also make a testing for the linkable label, and the result is the result is better than the Tsukamoto Fuzzy Inference System. We believe that, there must another testing about the relation of the number of linkable label and the result of the Tsukamoto Fuzzy Inference System. Our result make only 2 (two) linkable label of every label.

5. Conclusion

The conclusion that can be drawn from this study are as follows.

1. There is a limitation about Tsukamoto Fuzzy Inference System.
2. We can make a linkable label in the membership function of every label with the concept of Fuzzy Grid Partition.
3. The new membership function as the result of Fuzzy Grid Partition can correct the limitation of Tsukamoto Fuzzy Inference System.

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Author Profile:

Hartono received the Master degree in 2010 from the University of Putra Indonesia "YPTK" Padang, Indonesia in Computer Science and Bachelor Degree in 2008 from STMIK IBBI Medan, Indonesia in Computer Science. He is a lecturer at STMIK IBBI Medan. His current interests are in data mining and artificial intelligence. Nowadays, He Is a Student in a Doctoral Program in Computer Science at University of Sumatera Utara