NON LINEAR BEHAVIOR IN LEARNING PROCESSES

P. Manfredi (+) and V. R. Manfredi (*)

(+) Dipartimento di Scienze dell’Educazione, assegnista di ricerca e dottorando
(*) Dipartimento di Fisica dell'Università di Padova and INFN, Sezione di Padova

1. Introduction

This article is mainly based on R. E. Kahn's contribution to the book *Non Linear Dynamics in Human Behavior* [1].

As stressed by Bronowski [2], both in art and in science, a person becomes creative by finding "a new unity" that is a link between things which were not thought alike before. Indeed the creative mind is a mind that looks for unexpected likeness finding a more profound unity, a pattern behind chaotic phenomena (See Appendix 1). In the context of scientific discovery, it can also be argued that creativity is linked to a search in a space of hypotheses and a space of experiments. This "Dual Search" involves the formation of new hypotheses and new experiments which are then linked by a comparison of the prediciton derived from a hypothesis with the results obtained from the experiment.

Thus even after the creative mind has produced a new hypothesis, experiments follow which confirm or deny that initial insight.

Root-Bernstein [3] % has one of the characters in his dialogues suggest that the very nature of science involves a process of discovery in which:

"We actually transform ideas from one way of thinking to another.

We take numbers and turn them into pictures, or an apparatus and turn it into a mental abstraction. "Transformational thinking", call it. So the more different mental skills - tools of thought - you can utilize, and the more ways you can transform ideas, the better your chances of solving problems."

Thus both Bronowski and Root-Bernstein would seem to be arguing that there are creative processes applicable to both art and science in which a new idea is formed in the mind of a single individual. Following Kahn these processes can be called "chaotic thinking", which involves a movement toward attractors and their basins, especially cascading attractors when two or more attractors couple and blend together into a new self-organizing structure. However, to anchor this kind of thinking empirically, it is important for both scientists and artists to engage in Dual Search, as Klahr and Dunbar have suggested – to formulate and test new hypotheses with new experiments
[4,5].

It can be argued (and most scientists would) that such tests should take place "in simplified and tightly controlled laboratory simulations", rather than "in uncontrolled naturalistic situations". However, the problem remains of applying the laboratory research to specific real-life problems. Therefore, it is sensible to adopt Jenkins’s %

suggestion that: "The laboratory profits from the simulation of the world and real life problems as much as the real problems profit from the laboratory research" [6]. This balanced approach has the further advantage that creativity in learning is no longer seen as solely a cognitive process, but as a social practice in which the focus shifts "from the individual as learner to learning as participation in the social world": Learning then involves not simply "the acquisition of propositional knowledge" but more significantly, the construction of a new personal identity [7].

2. Chaos Theory in Learning

The fact that both the natural world and the brain are "inherently nonlinear" suggests that chaos theory might have an important role in learning [8,9]. However, the possible contribution of chaos theory to learning is still much disputed. Ennis has offered important reflections and sources for "reconceptualizing learning as a dynamic system", but she is concerned primarily with values as attractors and constraints in the educational system [10]. Doll sees chaos theory as central to future curriculum development in the context of post-modern structuralism, but, in the opinion of Kahn, is with the more basic issue of how competence, task difficulty and degree of awareness are related [11].

Cziko has argued that precisely because human behavior is so complex and unpredictable, researchers should be content to seek descriptions and interpretations, rather than predictions of behavior [12]. Certainly, in some of the applications of chaos theory to adult education and composition studies, the interest has been primarily in using chaos theory as a metaphor or aid to reflection at the descriptive level suggested by Cziko [13, 14]. However, there is increasing awareness that with the use of sophisticated non-linear statistics, chaos theory can be applied effectively to reading research, developmental disabilities, the nature of human action itself, and widely within the field of social psychology [15].

It is appropriate then to take some first steps in the application of chaos theory as a statistical tool to improve our understanding of the nature of learning and various attitudes to learning. Two approaches appear promising: (1) the logistic equation (See Appendix 2) and further applications of the work of Edward Lorenz [16] (See Appendix 3); and (2) a more basic systems theory approach seeking to understand the processes involved in the emergence of a dynamic system. Since Lorenz’s work is very much concerned with understanding the characteristics of a dynamic system, the line between these two approaches is at times blurred, especially as both are concerned with an analysis of process rather than structure. Whatever the particular statistical technique used to understand the chaotic phenomena, the dynamic system needs to be studied as "a system with a rule of evolution". As a first step, let us consider the learning process as analogous to the motion in the physical sciences: from one or more clearly delineated points, operations take place which draw the person or the physical system to a new position through a process which can be precisely described. While it is widely accepted that "chaotic motion is not a rare pehnomenon" [17], chaotic learning is not widely accepted. Yet the same necessary conditions for the existence of chaotic motion also apply to chaotic learning. As Baker and Gollub note:

"Several necessary conditions for chaotic motion are that (a) the system has at least three independent dynamical variables, and (b) the equations of motion contain a nonlinear term, that couples several of the variables … While these conditions do not guarantee chaos, they do make its existence possible [17]".

In brief, if the key variables and "rule of evolution" within the learning process can be clearly defined, then it should be possible to make use of chaos theory as a statistical aid in understanding the learning process.

In reflecting on the analogy of motion to learning, the work of Edward Lorenz is of particular assistance. In showing how weather patterns could be simulated in models based on twelve and then on only three equations, Lorenz "captured the essence of chaotic behavior in a wide range of natural systems". Furthermore, Lorenz later demonstrated the essence of chaos in one simple equation:

$$X_{new}=Rx_{old}\,(1-x_{old})$$

which possesses unexpectedly complicated behavior analogous to climate and weather. The equation works as follows: Choose a value of R and an initial value of x_{old}. Use these to calculate x_{new}. Then update x_{old} by letting it equal x_{new}. The procedure is then repeated as many times as you wish.

The equation contains the crucial physical ingredients of forcing, R; dissipation, (1-x); and nonlinearity (x^2) that make it a candidate for irregular or chaotic behavior. R could represent a temperature gradient in a dishpan experiment or a heating rate for a convecting fluid while x represents the energy or momentum of the fluid. The dissipation term, (1-x), keeps the solution bounded .

In the learning process what would be analogous to "forcing"?

Remembering that R could represent a heating rate which is forcing action in the fluid, in the context of learning it would be challenge – the relationship of task difficulty to competence – that is pushing an individual to increase prior knowledge. Completing the analogy, if x is the state of knowledge, then x_{old} represents the individual's content knowledge, which is being increased, so that the movement from x_{old} to x_{new} represents the learning process. Furthermore, the starting point of this dynamic learning system is x_{old} which represents the prior knowledge an individual brings to each learning challenge.

For example, in seeking to understand the reading process, many qualitative inventories have been developed that measure a student's prior knowledge and reading competence with texts of different degrees of difficulty [18]. Since these texts are graded by year for both task difficulty and reading competence, a vast amount of data is available that could be analyzed in the context of reading as a self-organized dynamic system for each student. In the context of

Lorenz's equation "1" would represent perfect knowledge; alternatively, a numerical system could be utilized in which a certain grade level (such as 10) could be set as a point at which the student would be judged to be a proficient reader.

While building upon the present linear analyses of reading, this Dynamic Model of Reading would have a number of advantages: (1) the importance of "teaching up" would be emphasized, so that teachers would be encouraged to give students texts that were one to two years above their independent reading levels; (2) the importance of relating task difficulty to competence would also be stressed, so that teachers would not give students texts that were much too difficult for their existing level of competence; (3) reading would be seen as a dynamic system in which prior knowledge, task difficulty and competence interacted within each student; (4) the feedback among these three variables could be studied in terms of how each student organized the act of reading as a personal self-organized system; and (5) the necessity of longitudinal studies would be emphasized in order to see how each student was progressing over time. Furthermore, particular stages in the reading process such as the initial act of "learning to read", as well as the later step of "reading to learn" could be analyzed in some detail, in order to clarify the feedback among the variables at different levels of competence.

There is increasing awareness that the language arts should be seen as a unity with reading, writing, listening, and speaking understood to be related variables in a dynamic system. Here again, the relationship of the variables could be studied using Lorenz's single equation, especially in terms of the feedback of the various measures old literacy upon each other. In a sense, the Dynamic Model of Reading proposed above is one possible implementation of Eiser's call for "a dynamic view of mind" based upon William James's awareness that "thought is in constant change" [19,20]. The recognition that reading and thinking can be taught together has already led to the development of an Independent Reading and Thinking Inventory which would provide a useful tool on which to build a Dynamic Model of reading.

The underlying idea in relating chaos theory to creativity is that literacy itself is "in motion"- that the movement from ignorance to knowledge can be defined in different way in different disciplines, but the basic pattern of movementhas an interdisciplinary foundation. The self-awareness of the individual is also crucial to the development of literacy, particularly because self-awareness of competence builds self-esteem. As the key variables in literacy development are defined, it should then be possible to build relevant one-dimensional maps which will identify the key attractors, constraints and boundaries [18]. One key hypothesis to be tested is whether awareness itself (or perhaps self-awareness of competence defined as self-esteem) is the crucial "forcing" agent in the development of literacy. In other words, when awareness is added as a further dimension to the development of literacy, the interaction between task difficulty, prior knowledge, competence, and self-awareness will provide an exciting formulation to understand and enhance literacy in each student. However, self-esteem should not be seen as a purely cognitive variable because of the relevance of emotional and evaluative factors.

To document these ideas for individual students is the next step, recording the sensitivity to initial conditions which characterizes all dynamic systems. For example, it should be possible to demonstrate the impact of changes in instructional techniques on learner motivation and achievement. Each student and each instructor will have a highly personal preference for different types of teaching and learning styles. It is important to recognize that such preferences will differ not only for each student, but often for each subject studied by a student, at changing levels of prior knowledge, task difficulty, levels of competence, and degrees of awareness of competence. Furthermore, it should be noted that this approach is not a plea for further development of the misleading Myers-Briggs Inventory with its various personality categories [21], but rather an assertion that the act of learning is itself part of a dynamic system, both within each person and in the interaction between student and teacher. To conceive of a person as having only one major personality type or learning style is dangerously simplistic when learning itself is such a dynamic system.

Of course, much work remains to be done in testing these ideas empirically, but respecting the individual student’s metacognitive awareness of the learning process should enable us to gather data about how each student is experiencing learning. It should be possible to determine how learning based upon instruction can draw an individual into self-regulated learning, since engagement itself becomes an attractor that transforms the individual from simply trying to understand the instructor into developing new ideas. The key step may be to link learning theory to connectionism- "the idea that many simple structures exhibit complex collective behavior because of connections between the structures" in a neural context [22].

Although these ideas do have a complexity which is not initially inviting, if learning itself is a dynamic system then complexity cannot be avoided. As H. L. Mencken once said, "For every complex problem, there is a simple solution- and it’s always wrong [23]. Creativity itself is a complex phenomenon. How much of this complexity is of the variety amenable to chaos theory is still an open question, but given the chaotic nature of the brain itself, the inherent complexity of the learning process is not in itself surprising. Perhaps the term "chaotic thinking" will prove to be a correct designation of how we learn to think creatively, and it may prove possible to improve our understanding of creativity and literacy through the use of nonlinear statistical models.

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Appendix 1

Chaotic Phenomena

Deterministic chaos denotes the irregular or chaotic motion that is generated by nonlinear systems whose dynamical laws uniquely determine the time evolution of a state of the system from a knowledge of its previous history. In recent years – due to new theoretical results, the availability of high speed computer and refined experimental techniques – it has become clear that this phenomenon is abundant in nature and has % far – reacing consequences in many branches of science (see the long list in Table 1 which is far from complete).

We note that nonlinearity is a necessary, but not a sufficient condition for the generation of chaotic motion.

Table 1: Some nonlinear systems which display deterministic chaos

Forced pendulum [1]
Fluid near the onset of turbulence [2]
Lasers [3]
Nonlinear optical devices [4]
Josephson junctions [5]
Chemical reactions [6]
Classical many-body systems (three-body problem) [7]
Particle accelerators [8]
Plasmas with interacting nonlinear waves [9]
Biological models for population dynamics [10]
Stimulated heart cells [11]

The observed chaotic behavior in time is neither due to external sources of noise (there are none in the Lorenz equations) nor to an infinite number of degrees of freedom (in Lorenz's system there are only three degrees of freedom) nor to the uncertainty associated with quantum mechanics (the systems considered are purely classical). The actual source of irregularity is the property of the nonlinear system of separating initially close trajectories exponentially fast in a bounded region of phase space (which is, e. g. , three-dimensional for Lorenz's system).

It becomes therefore practically impossible to predict the long behavior of these systems, because in practice one can only fix their initial conditions with finite accuracy, and errors increase exponentially fast. If one tries to solve such a nonlinear system on a computer, the result depends for longer and longer times on more and more digits in the (irrational) numbers which represent the initial conditions. Since the digits in irrational numbers (the rational numbers are of measure zero along the real axis) are irregularly distributed, the trajectory becomes chaotic.

Lorenz called this *sensitive dependence on the initial conditions* the butterfly effect, because the outcome of his equations (which describe also, in a crude sense, the flow of air in the earth's atmosphere, i. e. the problem of weather forecasting) could be changed by a butterfly flapping its wings. This also seems to be confirmed sometimes by daily experience.

The results described above immediately raise a number of fundamental questions:

- Can one predict (e. g. from the form of the corresponding differential equations) whether or not a given system will display deterministic chaos?
- Can one specify the notion of chaotic motion more mathematically and develop quantitative measures for it?
- What is the impact of these findings on different branches of physics?
- Does the existence of deterministic chaos imply the end of long-time predictability in physics for some nonlinear systems, or can one still learn something from a chaotic signal?

The last question really goes to the fundaments of physics, namely the problem of predictability. The shock which was associated with the discovery of deterministic chaos has therefore been compared to that which spread when it was found that quantum mechanics only allows statistical predictions.

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Appendix 2

The Logistic Map

This is a one-dimensional quadratic map defined by:

$$X_{n+1} = f_r(x_n) \equiv r\, x_n\, (1-x_n), \qquad (2.\ 1)$$

Where r is an external parameter.

The logistic map, the simplest nonlinear difference equation, appears in many contexts.

It had already been introduced in 1845 by P. F. Verhulst to simulate the growth of a population in a closed area. The number of species x_{n+1} in the year n+1 is proportional to the number in the previous year x_n and to the remaining area, which is diminished, proportionally to x_n, i. e.

$$X_{n+1} = r\, x_n\, (1-x_n)$$

where the parameter r depends on the fertility, the actual area of living, etc.

Another example is a savings account with a self-limiting rate of interest (Peitgen and Richter, 1984). Consider a deposit z_0, which grows with a rate of interest ε as $z_{n+1}=(1+\varepsilon)z_n=\ldots(1+\varepsilon)^{n+1} z_0$. To prohibit unlimited wealth, some politicians could suggest that the rate of interest should be reduced proportionally to z_n, i. e. $\varepsilon\rightarrow\varepsilon_0(1-z_n/z_{max})$. Then the account develops according to $z_{n+1}=[1+\varepsilon_0/z_{max}(1-z_n/z_{max})]z_n$ which becomes equal to eq. (2. 1) for $x_n=z_n\varepsilon_0/z_{max}(1+\varepsilon_0)$ and $r=1+\varepsilon_0$.

One could expect for both examples that due to the feedback mechanism the quantitites of interest (population and bank account) develop towards mean values. But as found by Grossmann and Thomae (1977), by Feigenbaum (1978), and by Coullet and Tresser (1978), and many others (see May, 1976, for earlier references) the iterates x_1,. $x_2\ldots$ of (2. 1) display, as a function of the external parameter r, a rather complicated behavior that becomes chaotic at large r's (see Fig.).

One can, therefore, understand the conclusion that May (1976) draws at the end of his article in "Nature":"Perhaps we would all be better, off not only in research and teaching, but also in everyday political and economical life, if more people would take into consideration that simple dynamical systems do not necessarily lead to simple dynamical behavior".

However, chaotic behavior is not tied to the special form of the logistic map. Feigenbaum has shown that the route to chaos that is found in the logistic map, the "Feigenbaum route", occurs (with certain restrictions) in all first-order difference equations $x_{n+1}=f(x_n)$ in which $f(x_n)$ has after a proper rescaling of x_n) only a single maximum in the unit interval $0 \le x_n \le 1$. It was found by Feigenbaum that the scaling behavior at the transition to chaos is governed by universal constants, the Feigenbaum constants α and δ, whose value depends only on the order of maximum (e. g. quadratic i. e. $f'(x_{max}) = 0$, $f''(x_{max}) < 0$, etc.). Because the conditions for the appearance of the Feigenbaum route are rather weak (it is practically sufficient that the Poincaré map of a system is approximately one-dimensional and has a single maximum), this route has been observed experimentally in many nonlinear systems.

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Appendix 3

Lorenz Model

Lorenz [1] considered the so called Rayleigh-Bernard effect in fluids. Assuming variations of the fluid to occur in only two spatial dimensions, Saltzman [2] had previously derived a set of first-order differential equations by expanding a suitable set of fluid variables in a double spatial Fourier series with coefficients depending on time. Substituting the expansion into the fluid equations results in an infinite set of coupled first-order ordinary differential equations. Truncation of this system by setting Fourier terms beyond a certain order to zero results in a finite-dimensional system which presumably yields an adequate approximation to the infinite-dimensional dynamics if the truncation is a sufficiently high order. To gain insight into the types of dynamics that are possible, Lorenz considered a truncation to just three variables. While this truncation is not of a high enough order to model the real fluid behavior faithfully, it was assumed that the resulting solutions would give an indication of the type of qualitative behavior of which the actual physical system was capable. The equations Lorenz considered were the following:

$$dX/dt=-\sigma X+\sigma Y,$$

$$dY/dt=-XZ+rX-Y,$$

$$dZ/dt=XY-bZ,$$

Where σ, r and b are dimensionless parameters. Referring to Figure 1. 4, the quantity X is proportional to the circulatory fluid flow velocity, Y characterizes the temperature difference between rising and falling fluid regions, and Z characterizes the distortion of the vertical temperature profile from its linear-with-height equilibrium variation. Lorenz numerically considered the case $\sigma=10$, $b=8/3$ and $r=28$. Taking the divergence of the phase space flow, we find that phase space volumes contract at an exponential rate of $(1+\sigma+b)=41/3$, $V(t)=V(0)\exp[-(41/3)t]$. It is this relatively rapid volume contraction which leads to the applicability of one-dimensional map dynamics to this problem.

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