How Game Theory can help to establish cost division in library consortia ?

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Outline

- Introduction: What is important?
- The cost-sharing problem and the game theory model
- Giving solutions and their properties
- An example: the CBUC case

What is important?

We look for cost-sharing rules:

- easy to understand
- easy to implement
- fair (¿?)
- consistent under renegotiation

The game theory model

- Game theory gives tools and define solutions to allocate the total cost of serving the members of a group demanding a common service

  N:set of members; C_N: total cost

- It takes into account the cost of serving a subgroup S of members

  C(S): cost of serving a subgroup S

Giving solutions:

- The core (Gillies, 1953)
- The constrained equal cost
- The constrained equal savings
- The talmudic rule (the Talmud is a vast collection of Jewish laws and traditions)

Giving solutions and their properties:
The Core

An allocation of the total cost C_N is a vector $y=(y_1, y_2, \ldots y_n)$ such that

$$\sum_S y_i = C_N$$

A cost allocation y is in the core if no subgroup S is charged more than its stand-alone cost

$$C(S) \geq \sum_S y_i, \quad \text{for all } S$$

Giving solutions and their properties:

The Core

If only information about individual stand-alone cost (c_i) is available, then

$$C(S) = \min\ [C_N, \textstyle\sum_S c_i]$$

Then, an allocation $(y_1,\ldots,y_n)$ of the cost is in the core if $\sum_N y_i = C_N$ and $0 \leq y_i \leq c_i$

Example: $(c_1, c_2, c_3, c_4) = (8,4,6,8)$ and $C_N = 10$

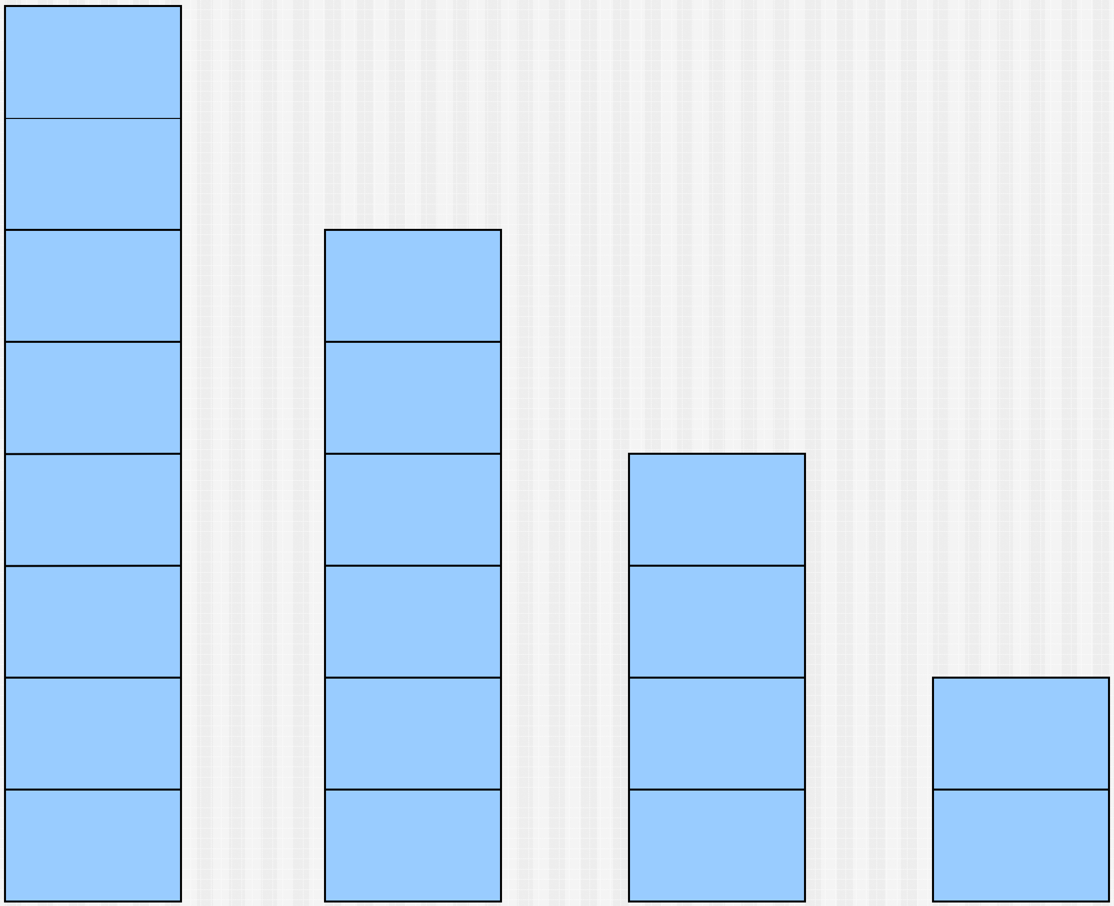
$y_1 + y_2 + y_3 + y_4 = 10$ and $0 \leq y_1 \leq 8$, $0 \leq y_2 \leq 6$,

$0 \leq y_3 \leq 4$, $0 \leq y_2 \leq 2$

Giving solutions and their properties:

The constrained equal cost

Agents share equally the cost under the condition that nobody is charged more than his stand-alone cost



$c_1 = 8$ $c_2 = 6$ $c_3 = 4$ $c_4 = 2$

C=20

C=17

C=18

C=14

C=6

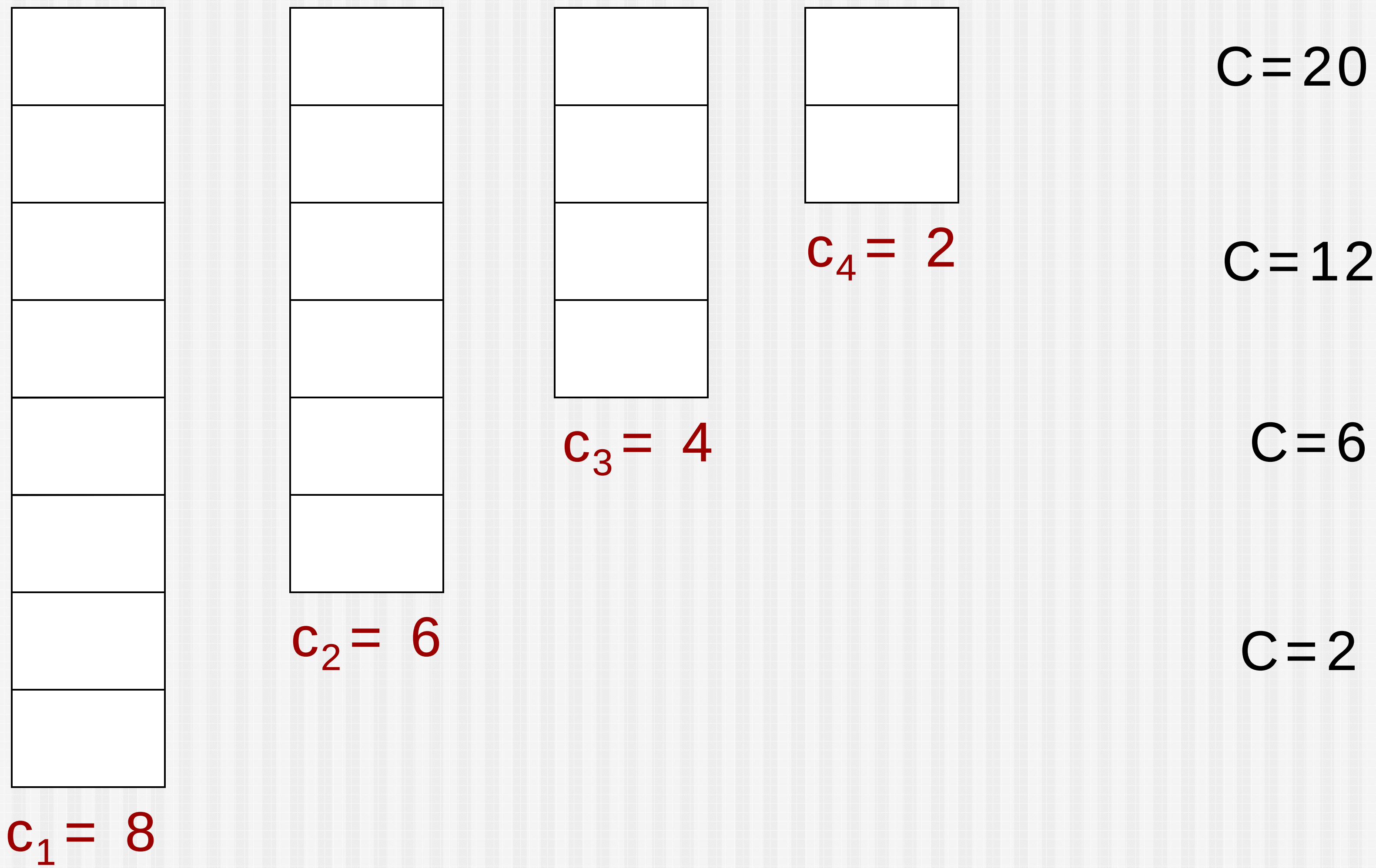
C=8

C=4

Giving solutions and their properties:

The Constrained equal savings

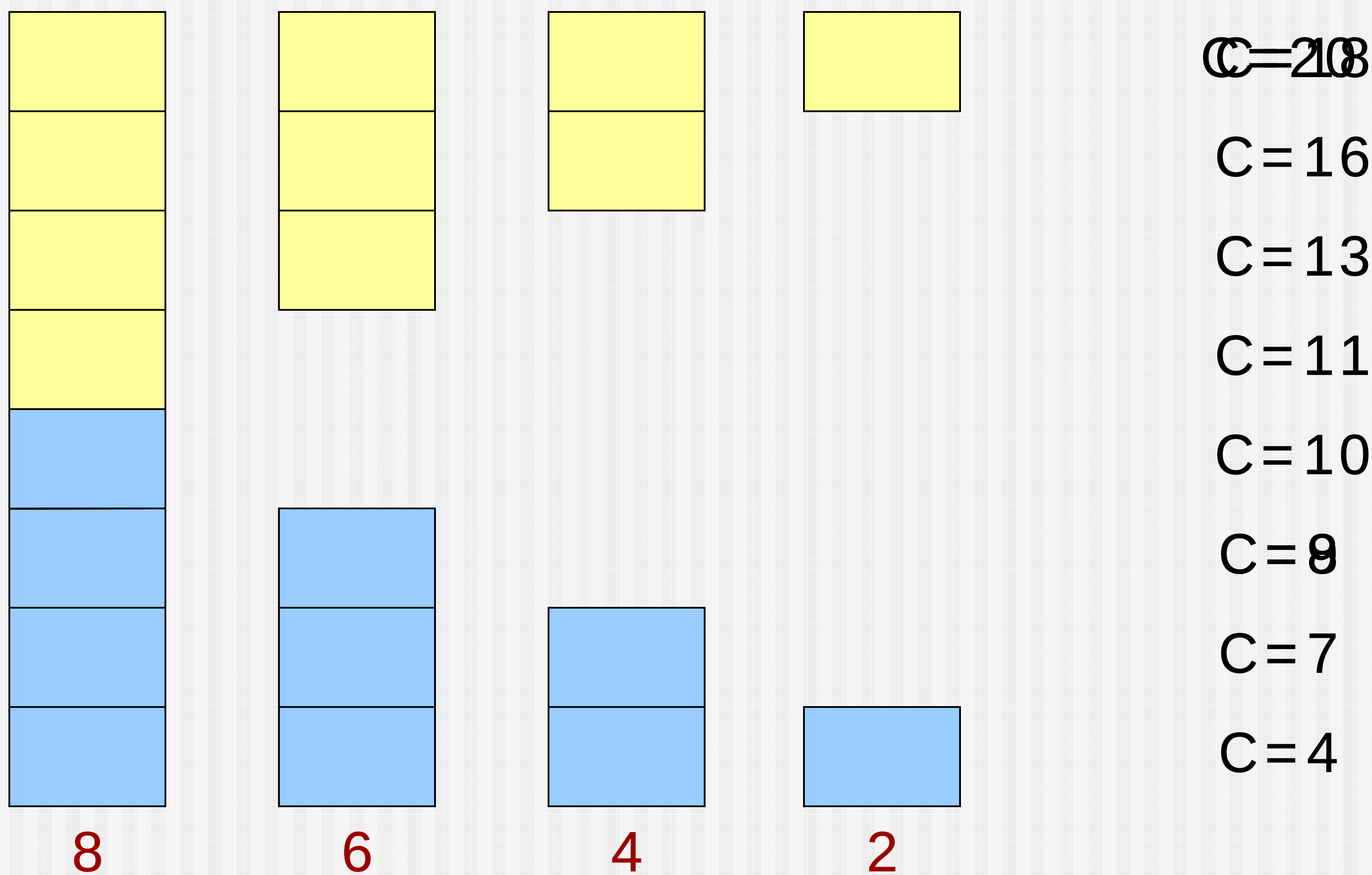
Agents get equal savings provided all agents end with a positive cost-share



Giving solutions and their properties:

The talmudic rule

Apply the CEC rule to half of the total cost and the CES rule to the other half



Giving solutions and their properties:

Comparing properties

(Herrero & Villar, 2001)

Prop / Rules	CEC	CES	Talmud.	Propor.
Equal Treatment	green	green	green	green (boxed)
Composition	green	green	red	green (boxed)
Consistency	green	green	green (boxed)	green
Comp. from separable cost	green	green	green (boxed)	red
Self-duality	red	red	green (boxed)	green (boxed)

An example: the CBUC case

MCB-Emerald (2001)

	Downloads	Stand alone	CEC	TL	CES	Prop
UOC	3667	18335	7210	15318,75	16113,13	12665,21
UB	2618	13090	7210	10073,75	10868,13	9042,14
UPC	1853	9265	7210	6248,75	7043,13	6399,95
UAB	1659	8295	7210	5278,75	6073,13	5729,91
UJI	958	4790	4790	2395	2568,13	3308,77
URV	750	3750	3750	1875	1528,13	2590,38
UPF	572	2860	2860	1430	638,13	1975,59
UDG	478	2390	2390	1195	168,13	1650,93
UDL	391	1955	1955	977,5	0,00	1350,45
UVIC	83	415	415	207,5	0,00	286,67
	13029	65145	45000	45000	45000	45000,00

d **5·d**

Remark: suppose each download is worth **5** €
and the total cost I **45.000** €

Final remarks

- Other game theory solutions

  The Shapley value (Shapley, 1953)
  The nucleolus (Schmeidler, 1969)
  The τ-value (Tijs, 1981)